LIMITATIONS OF THE RULING SPAN METHOD FOR OVERHEAD LINE CONDUCTORS AT HIGH OPERATING TEMPERATURES.

Report of the IEEE Task Force "Bare Conductor Sag at High Temperature".
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Abstract. This report summarises the work by the Task Force to review the accuracy of the ruling span method for conductors operated at high temperatures. The basics of the ruling span approximation method have been examined. The traditional ruling span approach can be used with little or no error for a typical overhead line crossing a rolling terrain to predict sags in suspension spans for conductor operating temperatures in the range of 50°C to 70°C. Sensitivity studies were performed using conductors "Lapwing" and "Tern" in order to quantify such ruling span assumptions as the effect of the longitudinal swing of suspension and line post insulators on conductor sags at high temperatures, and the effect of the suspension insulator string length on the equalization of conductor tensions in adjacent spans. Significant errors in estimating the sag at conductor temperature above 100°C may occur if the tension differences are not taken into consideration in line sections consisting of a series of spans of non-equal lengths. It was confirmed that the ruling span method is the most practical way to string conductors in multi-span line sections.

Key words: Conductor, ruling span, high temperature, sag, tension, insulator swing.

I. OBJECTIVE AND SCOPE.

The objective of this paper is to describe the widely accepted "ruling span" method of sag-tension calculation for multiple suspension spans between dead-end structures where the spans are nearly level but unequal of length. Errors due to operation of the conductor at high temperatures and due to imperfect tension equalization at supports is studied and several calculation corrections are noted.

In this paper, the high temperature operation means conductor temperature above 100°C (212°F).

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II. DEFINITION OF TERMS.

A = total conductor area
α = coefficient of elongation
D = conductor sag
D_r = ruling span sag
E = modulus of elasticity
H_0 = horizontal tension at initial conductor state
H = horizontal tension
L_0 = initial conductor length
L = conductor length
RS = "local" ruling span
S_r = ruling span length
S_i = suspension span length
T_0 = initial conductor temperature
T = conductor temperature
w = weight of conductor per unit length

III. THE RULING SPAN METHOD.

A. The basics.

The well known parabolic and hyperbolic equations defining the relationship between span, sag, and tension apply to single level dead-end spans. For a series of spans of unequal length and nearly level elevations, a simple method is needed to determine a theoretical level span length for which the sag and tension characteristics can be applied to determine the sag and tension behaviour of all spans. The solution of this problem was published in 1924 by E.S. Thayer, an electrical engineer in Seattle [1]. The solution is now called the Ruling Span method.

A common definition of ruling span is a level dead-end span that gives the same change in tension from changes in loading, creep, and/or temperature as that in a series of suspension spans between two dead-end structures [2]. This span "rules" the conductor's sag and tension behaviour for the line section. The ruling span method permits correct sagging of conductors and provides prediction of conductor behaviour with creep, loads and temperatures within the usual operating ranges of 50°C to 70°C.

The tension variations due to load or temperature changes will depend on the lengths of the spans in the section, and the section as a whole will react to load and temperature changes in the same way as a single "ruling" span [3]. It is a fictitious span with a rate of slack equal to the average rate of slack of the line section.

The equation for the ruling span length (S_r) of a line section of (S_i) suspension spans is:

$$S_r=\sqrt{\frac{\sum S_i^3}{\sum S_i}} \qquad (1)$$

and is referred in this paper as the traditional ruling span equation.

The parabolic approximation for the ruling span sag (D) is:

$$D_r=\frac{wS_r^2}{8H} \qquad (2)$$

The sag-tension behaviour of each of the spans in a line section is determined in the following manner:

- sag-tension calculations are made for a single dead-end span with length equal to the ruling span (Eq.1 and Eq.2).
- the tension in all of the suspension spans of the line section is assumed to be the same and equal to that of the ruling span under all loading and conductor temperature conditions.
- once the sag (D_r) of the ruling span has been calculated, the sag (D) of any other span (S) is calculated as:

$$D=\left(\frac{S}{S_r}\right)^2 D_r \qquad (3)$$

B. The assumptions.

The ruling span method is called an approximate method because of a number of unwritten assumptions made such as:

- span lengths are large compared to the difference in elevation of supports.
- the load per unit length is equal for all suspension spans in the line section.
- conductor temperature is the same along the line section.
- the suspension points between adjacent spans are free to move longitudinally without restraint. This is the fundamental assumption of the traditional ruling span method. When circumstances prevent or unduly restrict this free movement and tension equalization, sag predictions based on the ruling span method may be inaccurate.

Other errors resulting from the ruling span approach may be caused by:

- angle suspension insulators, running angle insulators and inverted "V" strings are neither true strain points (allowing no longitudinal movement of the attachment point) nor true suspension points (allowing unrestrained longitudinal movement of attachment points). Full tension equalization is unlikely at such points even for small longitudinal movements.
- at suspension structures supporting a large weight span, tension equalization may not occur even for modest longitudinal movement of the insulator attachment point.
- for post insulators, tension equalization depends on the combined flexibility of the suspension hardware (if any), the insulator, its attachment to the pole, and the pole.
- response of strain or suspension structures to varying loads. This can be significant, for example, when tubular steel structures or davit type arms are used.

C. The accuracy.

The ruling span approximation method may not be accurate enough to analyze the operation of a line, although it was used for the design of the line. This is especially true if there is a need to operate the line above the original design temperature.

Sag errors caused by incomplete tension equalization between suspension spans result in inaccurate calculations using Eq.1 and Eq.2. This is the main scope of this paper.

Sag errors caused by temperature variation along the line section generally cause lesser errors than those due to incomplete tension equalization, and it is outside of the scope of this Task Force.

Errors in sag estimates caused by the present methods of modeling of conductor's sag vs temperature relationship (which also affects the sags in individual dead-end spans) may be a future task of this Task Force.

While the Task Force certainly does not advocate discarding the existing methods of sag-tension calculations, it has identified situations where line engineers should be aware of the limitations of traditional calculation methods.

IV. SINGLE DEAD-END SPAN.

Sag-tension calculations can be complex even for single spans with fixed end points. The conductors' non-linear elasticity, thermal elongation, plastic creep elongation, and the various combinations of ice, wind, and temperature conditions may need to be considered.

Useful information can be obtained from the tension-temperature relationship in a level dead-end span, considering only the elastic properties and thermal expansion of the composite conductor, and making simplifying approximations.

The length (L) of conductor in span (S) can be calculated using the parabolic approximation, by:

$$L=S+\frac{w^2S^3}{24H^2} \qquad (4)$$

Arc elongation (or slack) is defined as the excess length of conductor (L) relative to the span length (S) and is given, in the parabolic approximation, by:

$$\frac{L-S}{S}=\frac{w^2S^2}{24H^2} \qquad (5)$$

The change in strain in a single dead-end span, in which the temperature changes from T_0 to T, accompanied by a change in tension from H_0 to H, is:

$$\frac{L-L_0}{L_0}=\frac{H-H_0}{AE}+\alpha\,(T-T_0) \qquad (6)$$

The "graphic method" of sag-tension calculation [4], assumes that the change of slack is equal to the change of strain. Using this assumption we can combine Eq.5 and Eq.6 to obtain Eq.7:

$$\frac{H-H_0}{AE}+\alpha\,(T-T_0)=\frac{w^2S^2}{24}\left[\frac{1}{H^2}-\frac{1}{H_0^2}\right] \qquad (7)$$

This cubic equation in H describes the approximate tension-temperature relationship for a single dead-end span. If H_0 and T_0 are known e.g., measured in field, the horizontal tension H may be computed for any given temperature.

V. MULTI-SPAN LINE SECTIONS.

Real overhead lines are not limited to spans with fixed end-point supports. In a typical transmission line, most spans are "suspension" whose end-point supports move, coupling each span with adjacent spans.

Transmission lines are usually sagged to maintain the insulators plumb. With temperature rise, creep and permanent strain from weather loads, the conductor elongates. When a line section has spans of differing lengths, the conductor elongation causes the insulators to depart from their vertical position. As temperature increases, the suspension point moves toward long spans and away from short spans to equalize horizontal tension. Figure 1, Appendix I, shows an insulator string and the forces acting at the suspension point. In general, at a suspension point between two spans, the movement of the suspension point caused by tension difference is restrained by the vertical load at the suspension point. The larger the load in the insulator string, the greater the restriction.

The ruling span method assumes that complete equalization is achieved, thus overstating the insulator swing. Suspension point movement is usually, but not always, less than that calculated using the ruling span method. Depending on the specific spans lengths, there may be a difference in horizontal tension between any two adjacent uneven spans.

The ruling span inaccuracies are largest for lines with short insulator strings [5], since the ruling span approximation assumes an infinite string length. The shorter the insulator's length, the greater is the restriction on movement. High operating temperatures (over 100°C) further degrades the accuracy of this approximation. A complete analysis of multi-span line sections should take into account conductor properties, spans, line profile, line angles, insulator string properties, support stiffness, original sagging and clamping-in procedures, weather loading history, creep, and a reasonably good knowledge of the existing condition of the line section under study.

VI. NUMERICAL EXAMPLES.

The basic assumption for the numerical examples is that the initial position of the insulators is vertical, either without or after offset clipping.

Case 1. Effect of Suspension Insulator Swing.

Consider a dead-ended line section consisting of 10 spans, conductor "Lapwing", ACSR, 45/7, total area = 1590 kcmil (1.249 in² or 805.8 mm²); weight=1.792 lb/ft (26.2 N/m), RTS=43780 lb (194.7 kN), $E_{al}=9.5*10^6$ psi ($65.5*10^3$ MPa), $\alpha=11.6*10^{-6}$ 1/°F ($21*10^{-6}$ 1/°C), $E_{st}=27.5*10^6$ psi ($189.6*10^3$ MPa). Initial condition: H_0=8440 lb (37.5 kN) @10°C; S_r=1000 ft (305 m); final condition: $T_{cond.}$ = 100°C. Suspension insulator string is 5 ft (1.52 m) long; and its weight is 120 lb (534 N).

Since the six computer programs used by the Task Force to calculate sag, tension, and swing at high temperatures showed very close results, Table 1, Appendix I, lists the average values of those calculations. As can be seen, the sag in the longest span of 1500 ft (457.2 m) is 4.9 ft (1.5 m) smaller when the tension differences are taken into account. In the span of 1150 ft (350.5 m), the sag @100°C is 1.4 ft (0.4 m) larger when the tension differences are considered. The explanation of these results is very important for line engineers and is given below.

It has been noted that short and long spans react differently to changes of temperature. Short spans are more sensitive to temperature changes than long spans. For the 1150 ft (350.5 m) span in our example, the positive sag error (when the actual sag is larger than the sag calculated using the ruling span method) depends on the span's length, the tension differences in adjacent spans, and insulator string lengths. Only for the idealized ruling span method, stiffness is independent of insulator string length. The tensions in the 1150 ft (350.5 m) span are modeled by the ruling span method to follow those of the 1000 ft (304.8 m) ruling span, but the restraints of the insulator strings cause the span to behave more like an 912 ft (278 m) span. The resulting behaviour is described later in this paper as a "local" ruling span which differs from the traditional ruling span method.

Case 2. Effect of line post insulators deflection.

Analysis using the case described below shows that the current practice can lead to sag errors, because although flexible, a polymer line post insulator is still a magnitude stiffer than a suspension insulator for a similar span.

Calculations were made using conductor "Tern", 795 kcmil (403 mm^2), ACSR, RTS=22100 lb (98.3 kN); w=0.8958 lb/ft (13.1 N/m); diameter = 1.063 in (27 mm); total area=0.6674 in^2 (430.6 mm^2). The ruling span = 500 ft (152.4 m). Initial tension is 15% RTS, i.e., 3315 lb (14.75 kN) @10°C. Line post insulator is 4.36 ft (1.33 m) long, and has a stiffness of approximately 2500 lb/ft (3728 kg/m). Table 2, Appendix I, shows the effect of deflection of line post insulators on sag at high temperatures.

The sag errors with the 2500 lb/ft line post insulators are approximately 1/2 of the difference between the ruling span and individual span cases. Stiffer line post insulators would cause the line section behave more like individual spans. For comparison, seven units insulator strings in the same span would have spring constant of about 120-150 lb/ft (1750-2190 N/m). The sag errors would be much larger if the spans lengths were increased.

Case 3. Effect of the insulator string length.

The basic assumption of the traditional ruling span method was verified i.e., as the insulator string length increases to infinity, the tension @100°C approaches the ruling span tension.

Calculations performed for Case I assumed an insulator string length of 5 ft (1.5 m). Similar calculations and comparison were made using the insulator string lengths of 2.5 ft (0.8 m), 14 ft (4.3 m), and 200 ft (61 m). The later can be considered as an insulator of infinite length.

At conductor temperature of 100°C, the 5 ft (1.5 m) long insulator string swing results in a tension difference of 182 lb (0.8 kN) between spans of 750 ft (228.6 m) and of 1150 ft (350.5 m). For the same conditions, the swing of the 2.5 ft (0.76 m) long insulator string results in tension difference of 339 lb (1.5 kN). For the assumed 200 ft (61 m) insulator string length, in the same spans, there is practically a complete tension equalization. This confirms the significant effect of the length of the insulators.

VII. OTHER APPROACHES TO RULING SPAN ERRORS.

High temperature sags can be modeled with alternative techniques such as "local" ruling span and fits to tension-temperature behaviour.

A better understanding of these techniques may be beneficial for the users of the real time line monitoring systems.

A. "Local" ruling span.

The concept of a "local" ruling span (RS) is to find a dead-end span that has the same tension-temperature relationship as each actual span. In order to define such a local ruling span, rewrite Eq.7:

$$RS=\sqrt{\frac{24\left[\alpha\left(T-T_0\right)+\frac{\left(H-H_0\right)}{AE}\right]}{\left(\frac{W}{H}\right)^2-\left(\frac{W}{H_0}\right)^2}} \qquad (8)$$

If (T_0, H_0) and (T, H) are known for a particular span, its "local" ruling span is obtained using this equation. This is a single-parameter fit to the tension-temperature relationship to two known points and is only valid to the accuracy to which the line section can be modeled (e.g. angle structures, elastic response of structures or uncertainties of elastic modulus and coefficient of thermal expansion). Alternatively, if (H_0) and (H) are determined by measuring the conductor tension directly [6] or derived from the measured sag at two known temperatures, an accurate single parameter fit can be established between the two fitted points. In both cases, the "local" ruling span's tension-temperature behaviour will differ from the actual ruling span except between and near the two fitted points. If necessary, this second order deviation can be calculated using a multi-span program and fitting "local" ruling spans for each span of interest. However, it may not be practical to have ruling spans which vary with temperature and location.

The "local" ruling span length was calculated using Eq.8 for Case 1. The calculation was performed for a temperature change from T_0=10°C to T=100°C, and the corresponding tension change is from H_0=8440 lb (37.54 kN) to H=5886 lb (26.18 kN).

$$RS=\sqrt{\frac{24\cdot\left[11.6\cdot10^{-6}\cdot(212-50)+\frac{(5886-8440)}{1.249\cdot9.5\cdot10^{6}}\right]}{\left(\frac{1.792}{5886}\right)^{2}-\left(\frac{1.792}{8440}\right)^{2}}}=912ft.$$

Because the "local" ruling span had the temperature-tension behaviour of a span shorter than 1000 ft (304.8 m), the sag in the 1150 ft (350.5 m) span was actually larger than that estimated using the traditional ruling span equation.

B. Fits to tension-temperature behaviour.

In general, as a result of the insulator swing at high operating temperatures the spans can interact in such a way that a multi-span sag-tension program may be necessary to predict the line section sag-temperature behaviour more accurately. In some cases it may be practical to run a multi-span program once and fit the results. One way to fit the tension-temperature of each span is to rewrite Eq.7 as:

$$T=a+bH+\frac{c}{H^{2}} \tag{9}$$

This equation is linear in the parameters a,b, and c, which can be fitted by linear regression to either the results from a multi-span sag-tension program or to observed values of (T) and (H). The fitting of (a) takes into account the constant terms involving (H_0) and (T_0); the fitting of (b) takes into account the "springiness" of all the other spans - either increasing or decreasing the effective spring constant of the conductor and; the fitting of (c) is similar to fitting a "local" ruling span. If (H) is required as a function of (T), either a closed-form solution of the cubic equation or an iterative solution may be used. Once (H) has been obtained for a given span, the sag may be computed using the actual span lengths and Eq.3.

VIII. CONCLUSIONS.

1. The traditional equations describing the relationship between temperature and span length, sag, and tension are fully valid for dead-end spans only. A multi-span line section can be analyzed as an equivalent single dead-end "ruling" span.
2. The traditional ruling span method can be used with acceptable error margins for lines which are operated below 100°C and have relatively equal and near level spans.
3. When old lines, originally rated for low operating temperatures, are uprated for operation at higher temperatures, the magnitude of sag errors should be evaluated using one of the available computer programs. The main source of errors is the longitudinal insulator swing in line sections with unequal spans.
4. For overhead lines planned for high temperature operation, it is recommended to add a buffer of about 1 m to the vertical clearance at maximum thermal sag.
5. Line post insulators can cause errors if the sag calculations are made using traditional ruling span equations when the span lengths vary significantly. In many cases, post insulators can cause individual spans to behave as if they were dead-ended at every structure.
6. The ruling span concept remains the most practical method to string overhead line conductors. The ruling span effects are not dependent on the type of conductor but rather on the amount of tension change per degree of conductor temperature change.

IX. ACKNOWLEDGMENTS.

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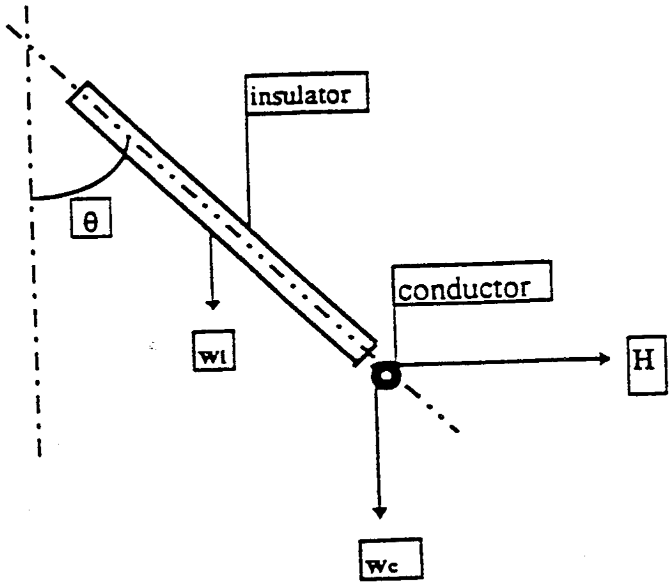
Appendix I.



$$\tan\theta = H/(w_l/2 + w_c),$$

where:

θ = angle of insulator swing

w_l = weight of insulator

w_c = total weight of a vertical span of conductor

H = total horizontal load including component of tension due to line angle

Fig. 1. Suspension Insulator Swing

Table I. Effect of Suspension Insulator Swing on High Temperature Sag.

S_r=1000 ft (304.8 m): @100 °C, sag=36.84 ft (11.23 m), tension=6090 lb (27.1 kN)										
span, ft	700	1150	750	450	900	750	950	1500	850	650
m	213.4	350.5	228.6	137.2	274.3	228.6	289.6	457.2	259.1	198.1
Sag @100°C based on S_r	18.1	48.8	20.7	7.5	29.8	20.7	33.3	83.1	26.6	15.6
	5.5	14.9	6.3	2.3	9.1	6.3	10.1	25.3	8.1	4.8
Sag @100°C w/swing effect	19.1	50.2	22.2	8.1	31.7	21.7	33.4	78.2	26.9	16.2
	5.8	15.3	6.8	2.5	9.7	6.6	10.2	23.8	8.2	4.9
Sag error	1.1	1.4	1.5	0.6	1.9	0.96	0.2	-4.9	0.3	0.6
	0.3	0.4	0.5	0.2	0.6	0.3	0.1	-1.5	0.1	0.2

Table II. Effect of Post Insulator Deflection on High Temperature Sag.

S_r=500 ft (152.4 m): @100 °C, sag=21.12 ft (6.44 m), tension=1328 lb (5.9 kN)										
span, ft	350	575	430	530	390	600	510	390	580	400
m	106.7	175.3	131.0	161.5	118.9	182.9	155.5	118.9	176.8	121.9
Sag@100°C based on S_r	7.3	19.8	11.1	16.8	9.1	21.6	15.6	9.1	20.2	9.6
	2.2	6.0	3.4	5.1	2.8	6.6	4.8	2.8	6.2	2.9
Sag@ 100°C w/deflection	8.6	19.2	11.6	16.8	9.8	20.1	15.5	9.9	19.4	10.5
	2.6	5.9	3.6	5.1	3.0	6.1	4.7	3.0	5.9	3.2
Sag@100°C individual span	9.4	18.2	12.3	16.2	10.8	19.3	15.4	10.8	18.4	11.1
	2.9	5.5	3.7	4.9	3.3	5.9	4.7	3.3	5.6	3.4
Sag error	1.2	-0.6	0.6	-0.09	0.66	-1.5	-0.12	0.7	-0.8	0.6
	0.4	-0.2	0.2	-0.03	0.20	-0.5	-0.04	0.2	-0.2	0.2